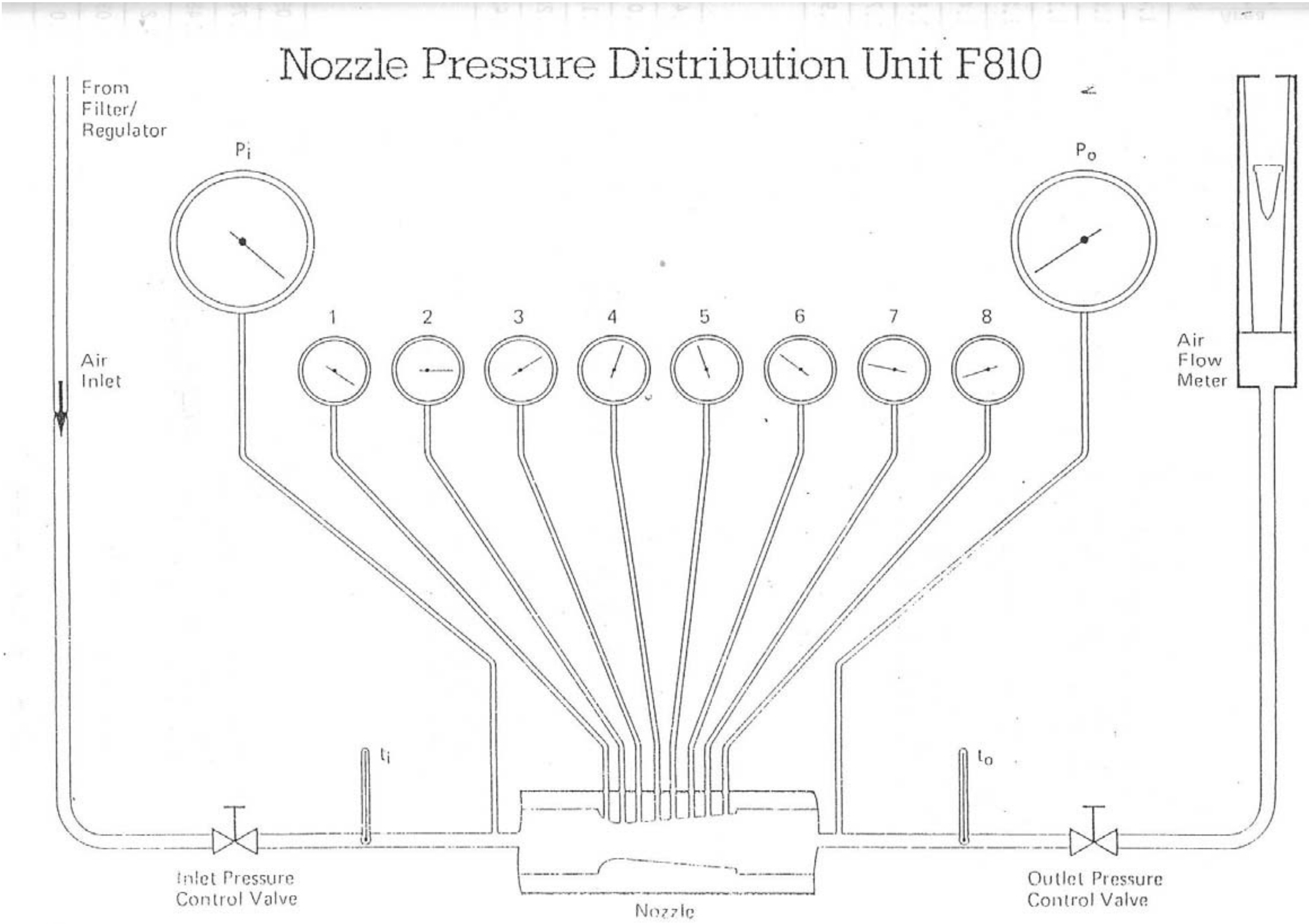
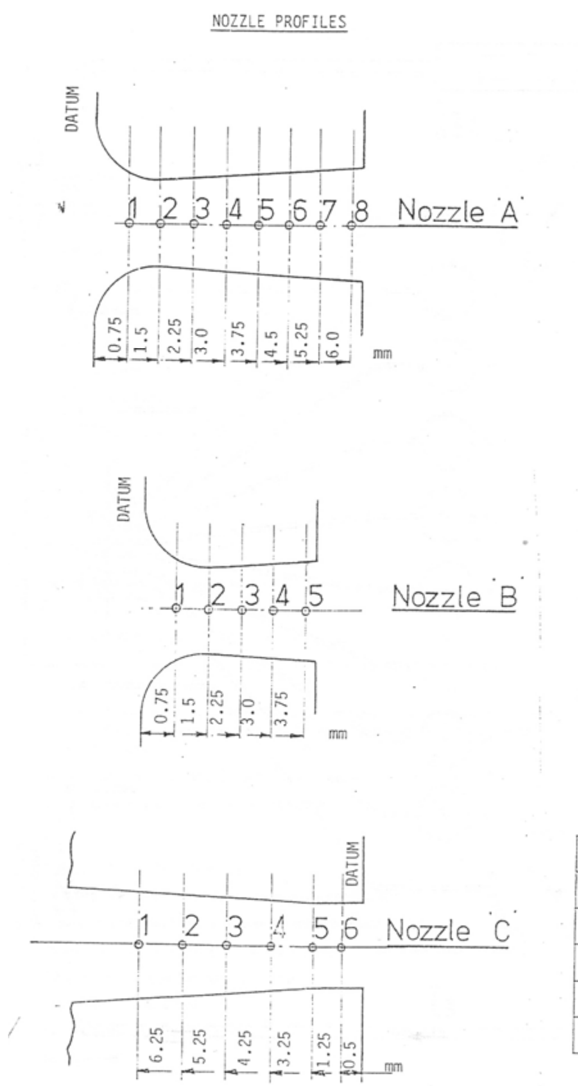
Nozzle pressure measurements

- The objectives are to measure the flow rates and pressure distributions within the converging and diverging nozzle under different exit and inlet pressure ratios.
- Analytic results will be used to contrast the measurements for the pressure and normal shock locations.

equipment setup



Nozzle types

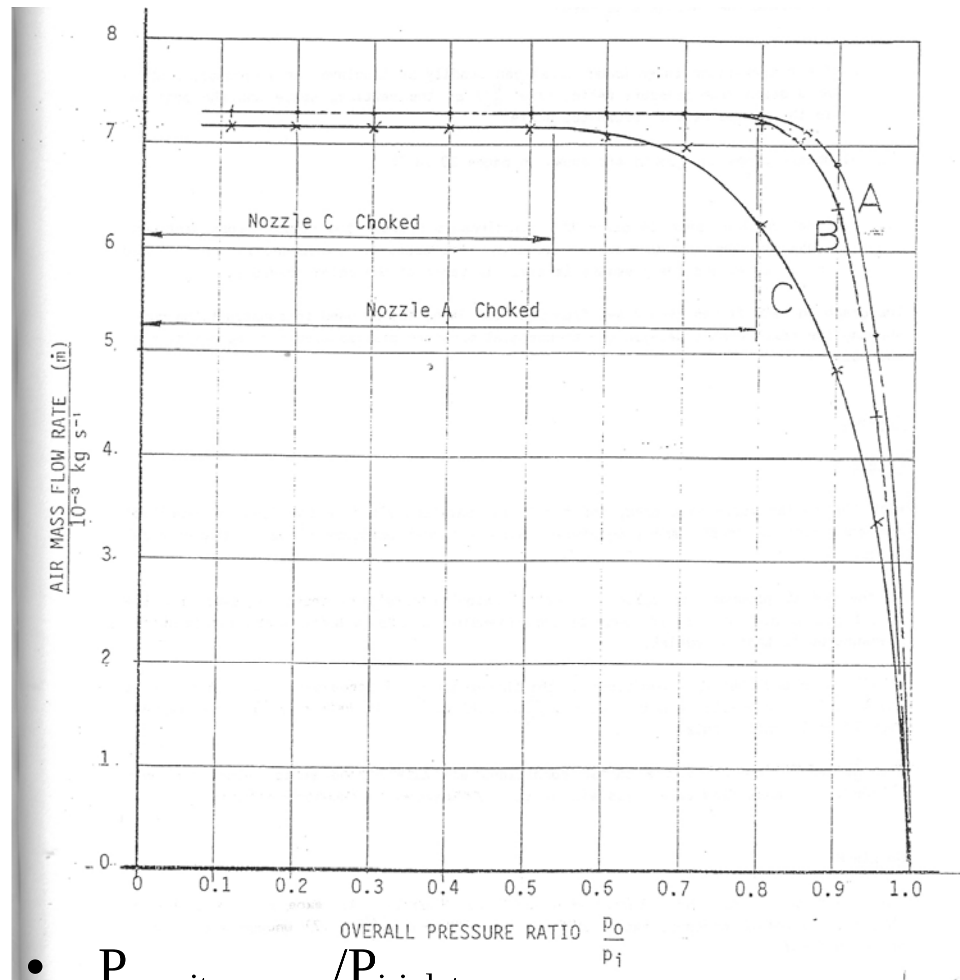


Tapping Point	Nominal Diameter mm	Area / Throat Area
1	2.4	1.44
2	2.0	1.0
3	2.13	1.13
4	2.26	1.28
5	2.39	1.42
6	2.52	1.59
7	2.66	1.77
8	2.79	1.94

1	2.4	1.44
2	2.0	1.0
3	2.13	1.13
4	2.26	1.28
5	2.39	1.42

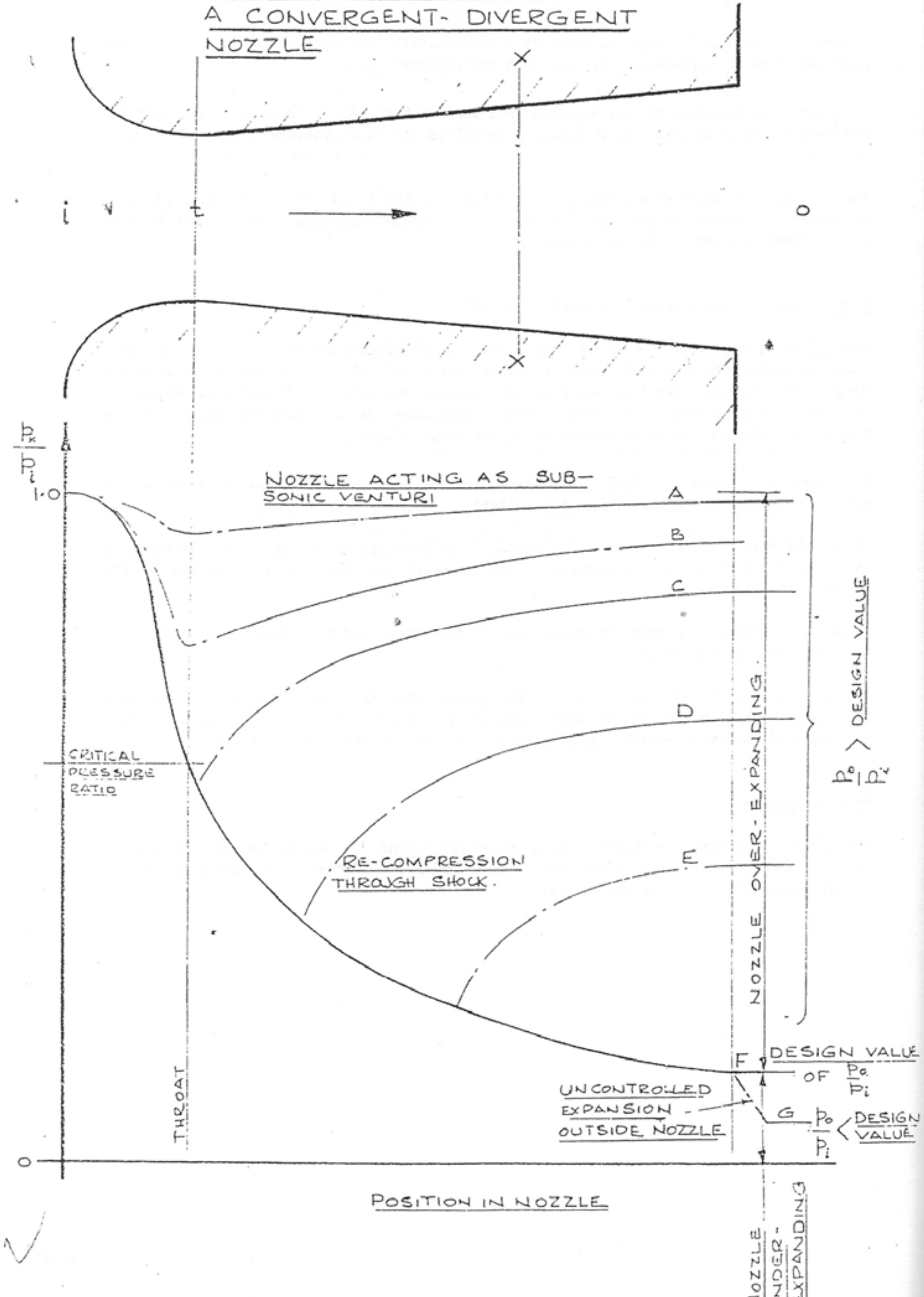
1	2.86	2.05
2	2.65	1.75
3	2.43	1.48
4	2.21	1.2
5	2.03	1.03
6	2.0	1.0

- Mass flow rate at different P_o/P_i

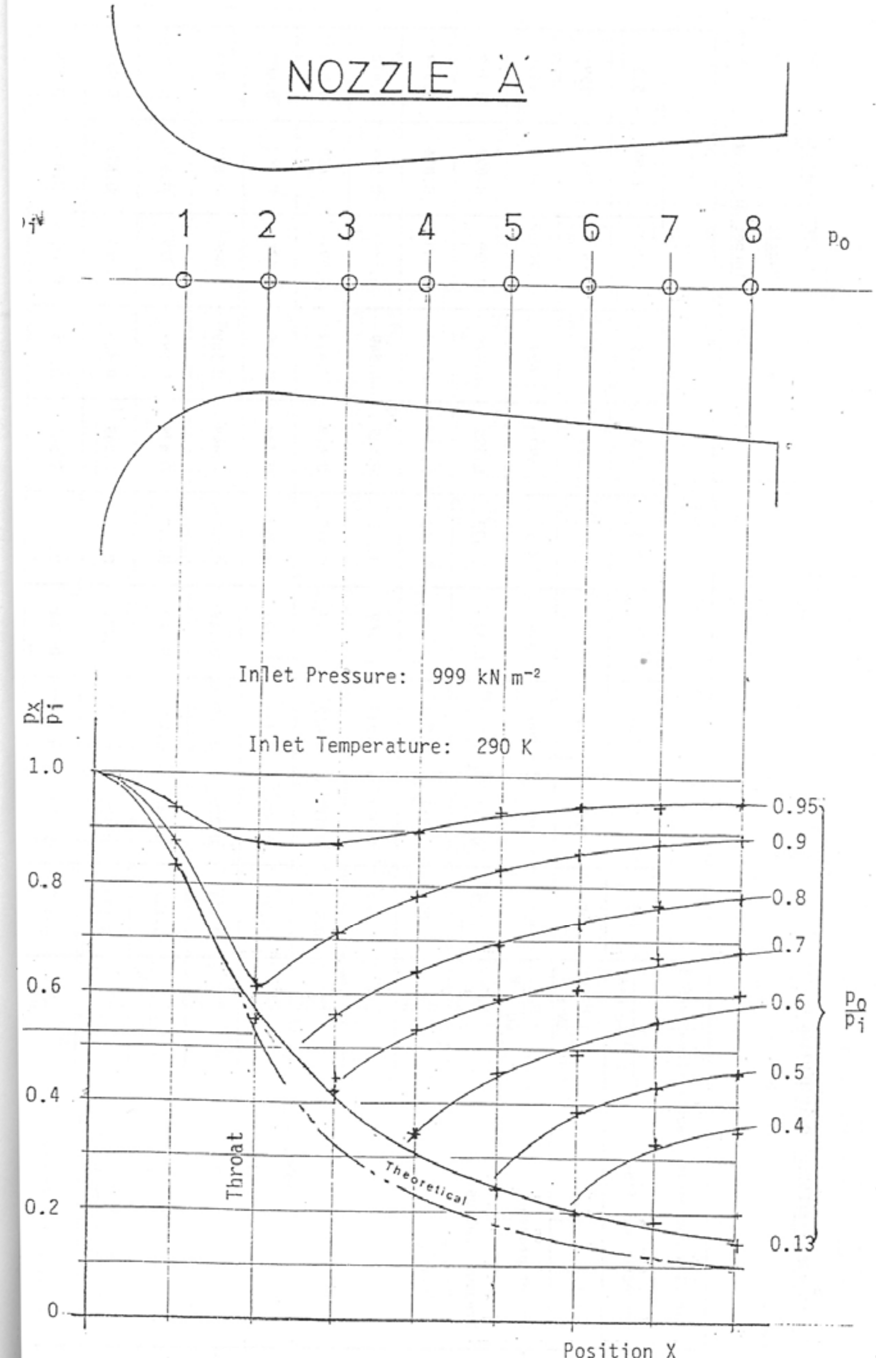


- $P_{o\text{-exit pressure}}/P_{i\text{-inlet pressure}}$

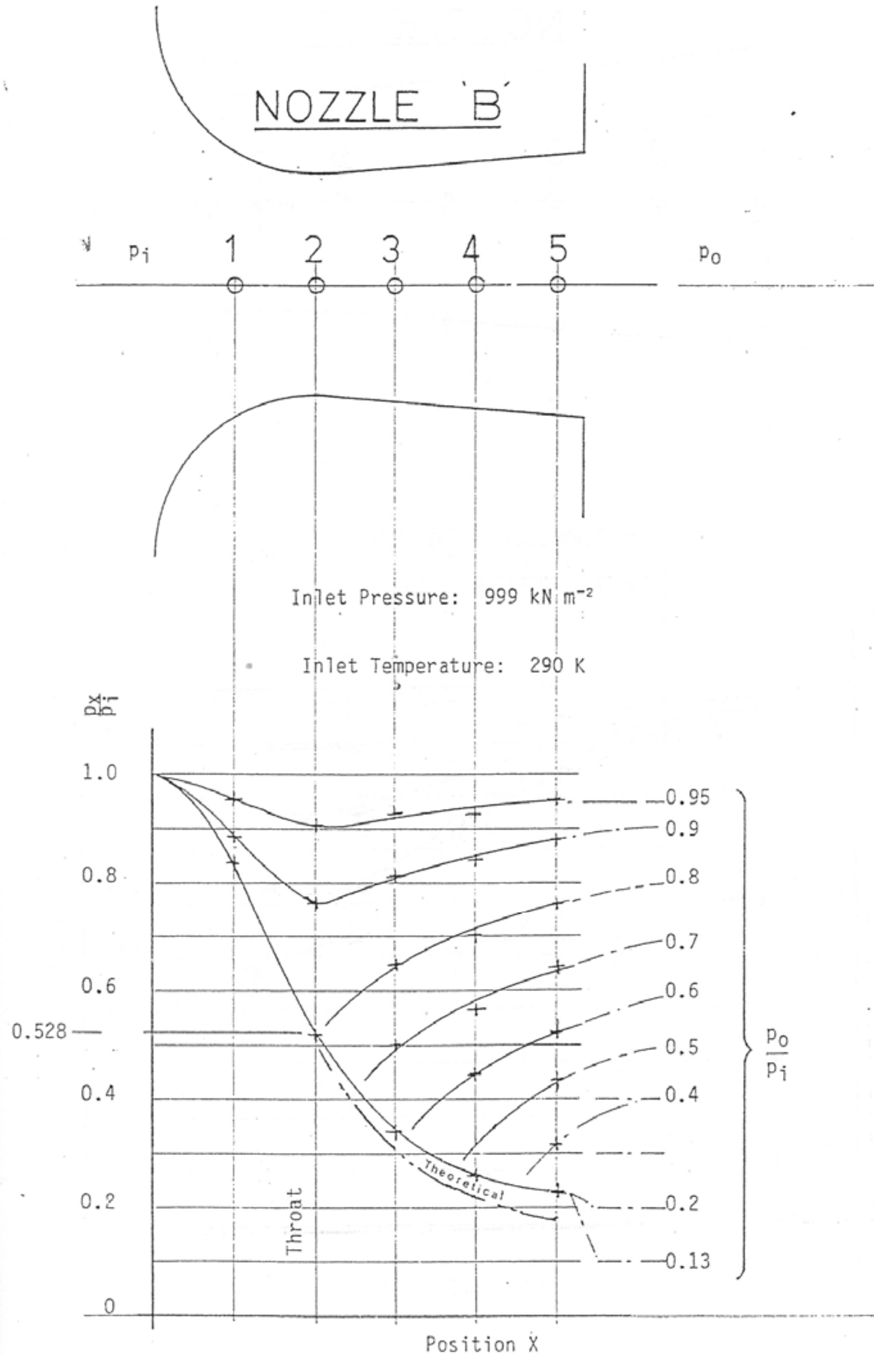
PRESSURE DISTRIBUTION IN A CONVERGENT-DIVERGENT NOZZLE

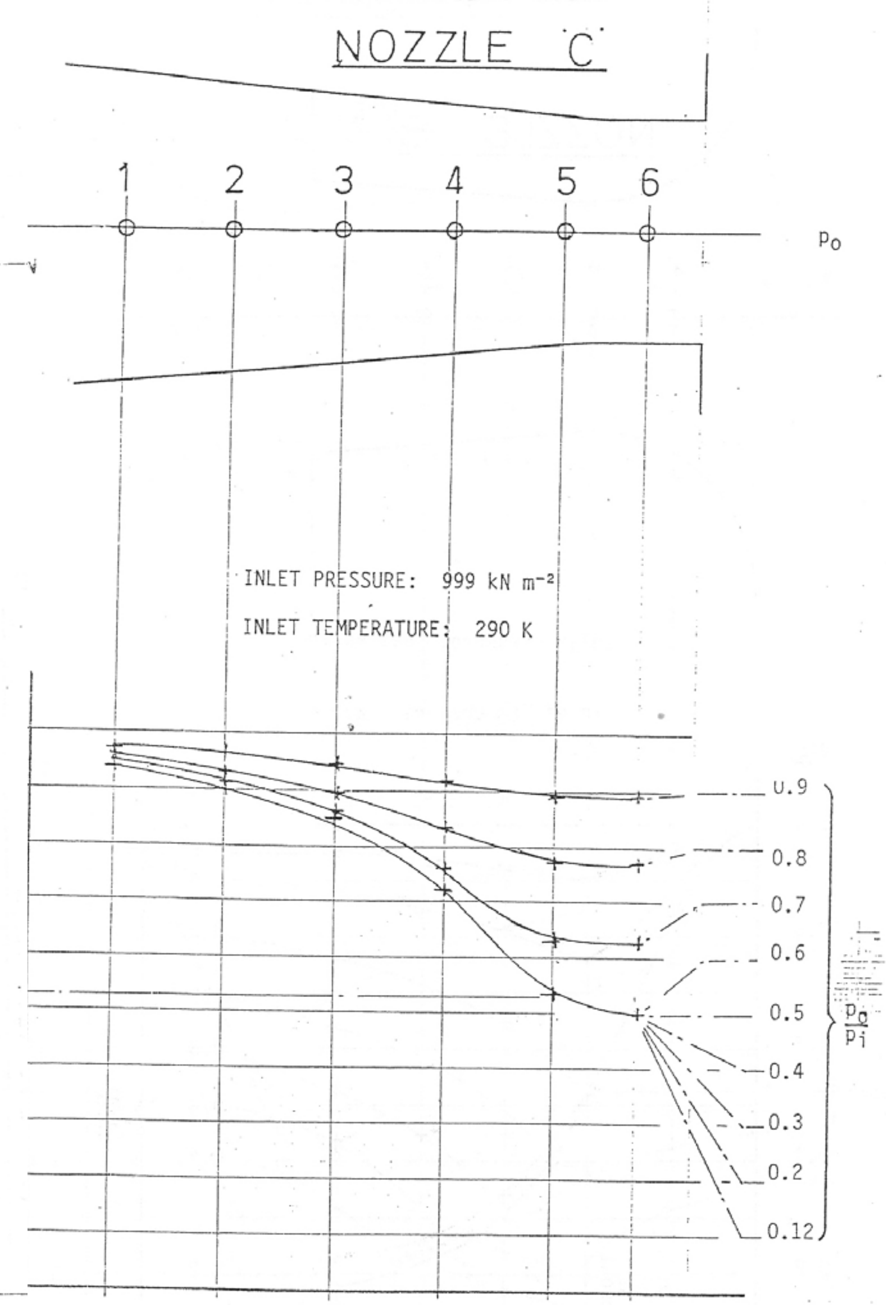


NOZZLE 'A'



EFFECT OF OVERALL PRESSURE RATIO ON THE "PRESSURE PROFILE"





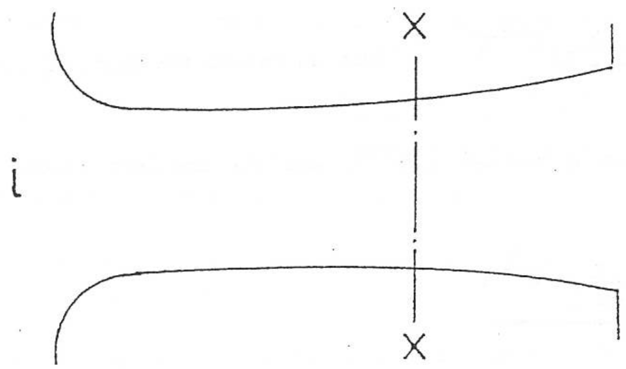
Final report

- Introduction
- Theory
- Experimental setup
- Measured and theoretical results
- Discussion
- Conclusion

APPENDIX

THEORY

Flow through a nozzle



For reversible and adiabatic one dimensional expansion through a passage, the following relationships apply at any section XX.

$$C_x = \sqrt{2(h_i - h_x)} \qquad \text{A}$$

$$A_x = \frac{\dot{m} v_x}{C_x} \qquad \text{B}$$

If it is assumed that the relationship between p and v in such an expansion is pv^k = constant,

$$h_i - h_x = \int_i^x v\,dp$$

$$= \frac{k}{k-1} p_i v_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{k-1}{k}}\right] \qquad \text{C}$$

and substituting this in A,

$$C_x = \sqrt{\frac{2k}{k-1} p_i v_i \left[1 - \frac{p_x}{p_i}^{\frac{k-1}{k}}\right]} \qquad \text{D}$$

Also, $$v_x = \left(\frac{p_i}{p_x}\right)^{\frac{1}{k}} v_i$$

and, substituting for C_x and v_x in B,

$$A_x = \frac{\dot{m} \frac{p_i}{p_x}^{\frac{1}{k}} v_i}{\sqrt{\frac{2k}{k-1} p_i v_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{k-1}{k}}\right]}}$$

Rearranging,

$$\frac{\dot{m}}{A_x} = \sqrt{\frac{2k}{k-1} \frac{p_i}{v_i}\left[\left(\frac{p_x}{p_i}\right)^{\frac{2}{k}} - \left(\frac{p_x}{p_i}\right)^{\frac{k+1}{k}}\right]} \qquad \text{E}$$

This expression has a maximum value, when

$$\frac{p_x}{p_i} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$ (This is called the <u>Critical Pressure Ratio</u>)

Since $\dot{m}$ is a constant, $\frac{\dot{m}}{A_x}$ is a maximum when A_x has its smallest value, i.e. at the "throat".

Thus,

$$\underline{\underline{\frac{p_t}{p_i} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}}} \qquad \text{F}$$

If this value is substituted in D,

$$C_t = \sqrt{\frac{2k}{k+1} p_i v_i} \qquad \text{G}$$

or $C_t = \sqrt{k\, p_t v_t}$ which is the <u>local speed of sound</u>.

Substituting the throat conditions into equation B, i.e.

$$C_t = \sqrt{\frac{2k}{k+1} p_i v_i}$$

and

$$v_t = \left(\frac{2}{k+1}\right)^{\frac{-1}{k-1}} v_i$$

we obtain

$$\underline{\underline{\frac{\dot{m}}{A_t} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{2k}{k+1} \frac{p_i}{v_i}}}} \qquad \text{H}$$

If Equation H is divided by E we obtain the relationship between the throat area and the area at any section x.

$$\underline{\underline{\frac{A_x}{A_t} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{k-1}{(k+1)\left[\left(\frac{p_x}{p_i}\right)^{\frac{2}{k}} - \left(\frac{p_x}{p_i}\right)^{\frac{k+1}{k}}\right]}}}} \qquad \text{J}$$

and for nozzle exit,

$$\frac{A_o}{A_t} = \left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} \sqrt{\frac{k-1}{(k+1)\left[\left(\frac{p_o}{p_i}\right)^{\frac{2}{k}} - \left(\frac{p_o}{p_i}\right)^{\frac{k+1}{k}}\right]}} \qquad \text{K}$$

From the foregoing it will be seen that when a compressible fluid expands reversibly and adiabatically in a passage through a pressure ratio

$$\frac{p_o}{p_i} < \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

(i) The passage must have reducing cross-sectional area, i.e. converge, until

$$\frac{p_t}{p_i} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}}$$

and must then have increasing cross-sectional area, i.e. diverge.

(ii) At the minimum area in the passage, i.e. the throat, the velocity of the fluid is the local speed of sound.

(iii) In the converging portion, velocity will be subsonic and in the diverging portion will be supersonic.

(iv) The mass flow rate through the passage is determined by the cross-sectional area of the throat and the properties of the fluid at inlet. It is not affected by the value of p_2 as long as the critical pressure ratio is maintained at the throat.

If the fluid flowing is a perfect gas we may use the following relationships:

$$p_v = RT$$

$$R = C_p - C_v$$

$$\gamma = \frac{C_p}{C_v}$$

$$h_1 - h_2 = C_p(T_1 - T_2)$$

and during a reversible and adiabatic process $p_v\,\gamma =$ Const, etc.

For air at the conditions met in this unit, we may assume that it behaves as a perfect gas with

$$R = 287.1 \quad J\ kg^{-1}\ K^{-1}$$

$$\gamma = 1.4$$

$$C_p = 1005 \quad J\ kg^{-1}\ K^{-1}$$

Using these relationships, Equation C becomes

$$C_x = \sqrt{\frac{2\gamma}{\gamma - 1} RT_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{\gamma - 1}{\gamma}}\right]}$$

$$= \sqrt{\frac{2 \times 1.4 \times 287.1}{0.4}} \sqrt{T_i \left[1 - \left(\frac{p_x}{p_i}\right)^{\frac{0.4}{1.4}}\right]}$$

$$C_x = 44.83 \sqrt{T_i \left[1 - \left(\frac{p_x}{p_i}\right)^{0.286}\right]} \quad m\ s^{-1}$$

L

Equation H becomes

$$\dot{m} = A_t \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma - 1}} \sqrt{\frac{2\gamma}{\gamma + 1} \; \frac{p_i^{\,2}}{RT_1}}$$

$$= A_t \left(\frac{2}{2.4}\right)^{\frac{1}{0.4}} \sqrt{\frac{2 \times 1.4}{2.4 \times 287.1} \quad \frac{p_i}{T_i}}$$

$$\underline{\underline{\dot{m} = 0.0404 \sqrt{\frac{A_t p_i}{T_i}}}} \quad \text{kg s}^{-1} \qquad \text{M}$$

Equation G becomes

$$C_t = \sqrt{\frac{2\gamma}{\gamma + 1} \; RT_i}$$

$$= \sqrt{\frac{2 \times 1.4 \times 287.1}{2.4} \times T_i}$$

$$C_t = \underline{\underline{18.3 \sqrt{T_i}}} \quad \text{m s}^{-1} \qquad \text{N}$$

It should be appreciated that the above relationships apply to reversible and adiabatic, i.e. isentropic, flow.

Although flow through practical nozzles may usually be assumed to be adiabatic, there will be various losses due to friction and shock - particularly in the divergent portion - which will render the expansion irreversible.